Using the so-called chimney in the Bent Pyramid to measure solar elevations in summer

Kyle Weyburne

ABSTRACT

The so-called "chimney" is a deep, vertical shaft located in the centre of the Bent Pyramid that starts at ground level and descends for over 15 metres into the earth. While the purpose of this chimney is disputed in scholarly literature, this paper offers the speculative hypothesis that it served as a tool for solar observations. For a period of time (before the pyramid was built above it), the shaft would have been open to the elements, allowing the sun's light to reach the bottom corner at the summer solstice, and thereby dividing this structure into two equal, but inverse triangles: one filled with light, the other dark. This precision suggests that the chimney may have been specifically designed to enable this phenomenon to occur. Moreover, further investigation shows that, for a period of 47 days centred around the solstice, the sun created a line of light on the floor of the chimney that would have made it possible to accurately measure solar elevations.

KEYWORDS

chimney – Snefru – Bent Pyramid – solar elevations – Rhind Papyrus

استخدام ما يسمى بالمدخنة بالهرم المنحنى لقياس الارتفاعات الشمسية فى الصيف

كايل ويبورن

ملخص

إن ما يسمى «المدخنة» هو عبارة عن بئر رأسية عميقة تقع فى منتصف الهرم المنحنى، وهى تبدأ من مستوى الأرض وتهبط بعمق لأكثر من 15 متراً أسفل سطح الأرض. فى حين أن الغرض من هذه المدخنة محل خلاف حتى الآن فى الكتابات الأثرية العلمية، تقدم هذه الورقة فرضية تخمينية بأن هذه البئر كانت بمثابة أداة للرصد الشمسى. فخلال فترة من الزمن (قبل بناء الهرم فوقها)، كانت البئر مفتوحة لا يعلوها شىء، مما يسمح لضوء الشمس بالوصول إلى الركن السفلى منها خلال الانقلاب الصيفى، وبالتالى تقسم البئر إلى مثلثين متساويين، ولكنهما معكوسان: أحدهما يملؤه النور والآخر مظلم. تشير هذه الدقة إلى أن المدخنة ربما كانت مصممة خصيصاً لتمكين حدوث هذه الظاهرة. وبالإضافة إلى ذلك، أظهرت المزيد من الأبحاث، أنه خلال مدة 47 يوماً تتركز حول الانقلاب الشمسى، كونت الشمس خطاً من الضوء على أرضية المدخنة كان من شأنه أن يجعل من الممكن قياس الارتفاعات الشمسية بدقة.

الكلمات الدالة

مدخنة – سنفرو – الهرم المنحنى – ارتفاعات شمسية – بردية رين**د**

The Bent Pyramid at Dahshur was built in the 26th century BC by King Snefru.[1] It has two separate internal sections, one above ground, the other below. The one below ground is entered from the northern side of the pyramid and is comprised of a chamber, an antechamber, and an unusual vertical shaft which archaeologists have dubbed the "chimney". The chimney has an intricate design and is linked directly to the chamber by a doorway, as well as by a horizontal shaft called the "window". The purpose of this chimney is unclear, and we lack any contemporary documentary evidence about its origin or intended use. However, the structure and design of the chimney itself may offer clues to its purpose that prior scholars have missed. Based on a preliminary analysis of the dimensions and positioning of the shaft, therefore, this article offers a speculative hypothesis that the chimney could have been designed to allow solar observations before the pyramid was built above it.

The chimney is a vertical shaft which begins at ground level and descends for over 15 metres. At its base, the dimensions of the chimney are 90 × 153 cm.[2] Its general form is of a rectangular shaft which rises upwards from there in two distinct stages, in which the form of the bottom half is repeated in reverse in the top half. Corbelling commences on the west and east sides at a height of 393 cm. The shaft juts outwards on both sides at this mark, though this is not symmetrical, as it juts out further on the west side. Then, the corbelling staggers inwards at an even rate on both sides, but since the east side does not jut out as far as the west, it takes just two laps for it to return to being in line with the wall (at the 620 cm mark), but requires four laps for the west side to return to being in line with the wall (at the 765 cm mark). The shaft rises upwards from there in its original, rectangular shape, before the corbelled design repeats at 1057 cm. This time, however, the pattern is reversed; the east side juts out further this time, and so requires four laps to return to the straight line, whereas the west requires just two. The chimney terminates at 1527 cm (see fig. 1b).

THEORIES ON THE PURPOSE OF THE CHIMNEY

The dimensions of the chimney are very unusual; it is a cramped space that rises over 15 metres high and presently leads nowhere. To date, no theory has been forwarded that, to my mind, adequately explains the chimney's dimensions and purpose. The most dominant theory has been presented by Gilles Dormion and Jean-Yves Verd'hurt (2016). They argued that the chimney was originally to have provided a vertical path linking the lower chamber to the upper chamber: "Ce cheminement vertical fut, toutefois, condamné et suggère que l'ensemble du dispositif inférieur a été abandonné au bénéfice de l'appartement supérieur" (Dormion – Verd'hurt 2016: 7).[3] According to Frank Monnier and Alexander Puchkov, the upper chamber was the intended burial chamber, while the lower chamber was simply an antechamber. In their opinion, "the 'chimney' was without doubt initially intended to lead to a burial chamber… The architect probably judged that the 'chimney' was too perilous to allow access to the funeral procession, and so an alternative arrangement had to be found" (Monnier – Puchkov

1 Jürgen von Beckerath estimates that the reign of King Snefru commenced *ca.* 2575 BC, while Erik Hornung proposes *ca.* 2573 BC +/- 5 years or 2548 BC +/- 5 years (see Beckerath 1997; Hornung – Krauss – Warburton 2006: 486).

2 All measurements of the chimney are sourced from Maragioglio – Rinaldi (1964: tav. 12.1).

3 In their opinion, the dead end at the top of the chimney is indicative of its abandonment.

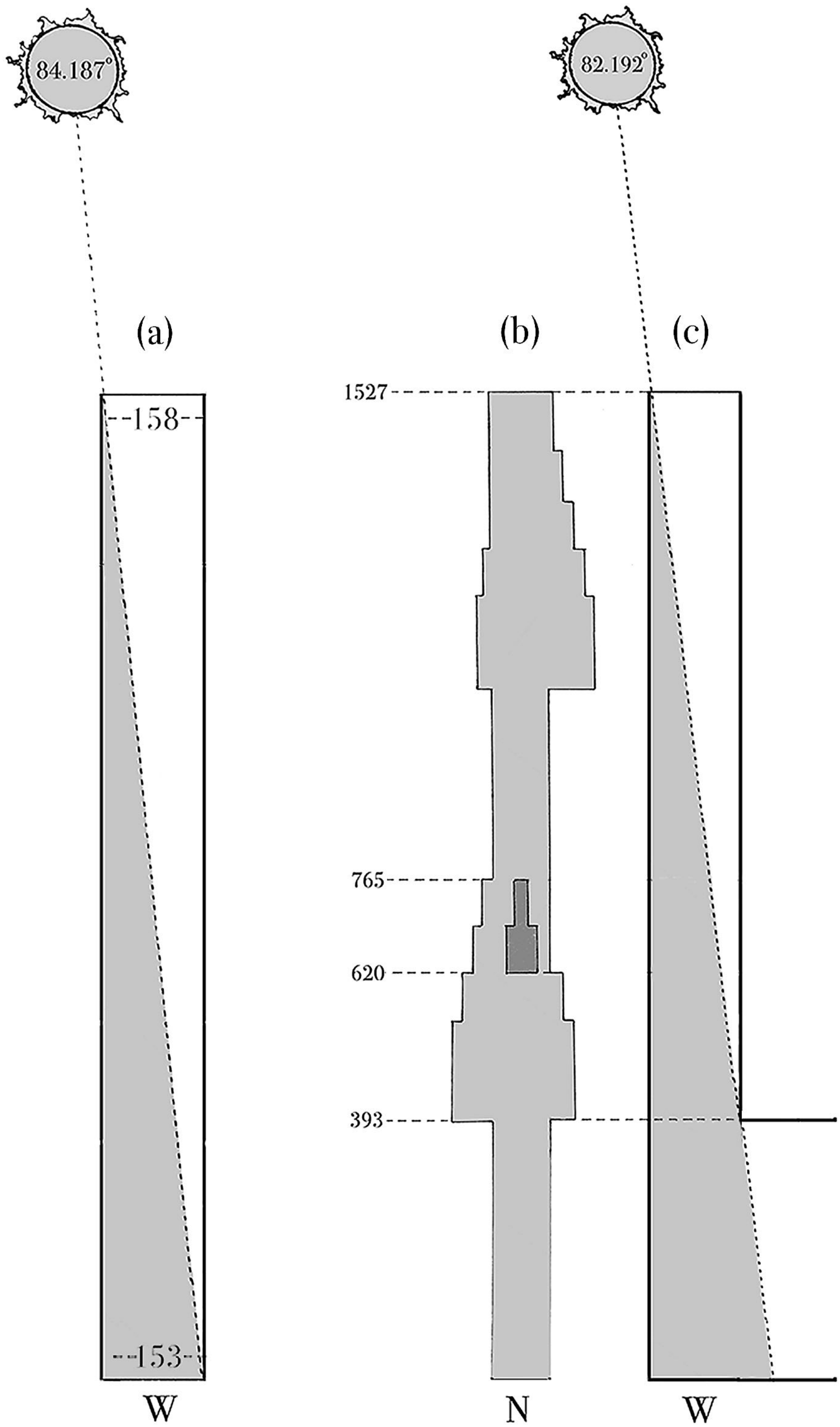

Fig. 1 a) The chimney looking west with the light cast by the noonday summer solstice sun; b) The chimney, looking north (on the left); c) The chimney looking west with passageway on the right, showing the minimum solar elevation required for the sun's light to clear the lintel and reach the floor of the passageway (drawing K. Weyburne)

2016: 21). The scenario presented by Monnier and Puchkov was that the architect had a sudden realisation that access to the chamber during the funerary procession would be too difficult via the chimney and so it was abandoned. The problem with this theory is that it would have been the architect who supposedly designed this shaft to be so cramped and so high, making it difficult to imagine that funerary access was ever the motivation.

One reason given by Monnier and Puchkov for their abandonment theory is that the two portcullis stones which were designed to seal the chimney at two different heights, were found opened (Monnier – Puchkov 2016: 21). This however does not mean that they were left open by the builders; instead, they may have been opened by intruders. This is supported by a reading of Vito Maragioglio and Celeste Rinaldi, who noted that the window which connects the lower chamber to the chimney was expanded at some point. Currently the window has a corbelled arrangement spanning three courses of brickwork, with its width decreasing with each new course line, but Maragioglio and Rinaldi designate the bottom course (the widest course, which is comprised of a single stone block) as "blocco mancanti" (Maragioglio – Rinaldi 1964: tav. 12.1). The original width of the window was only 57 cm.[4] This is a somewhat tight fit for an adult. Treasure hunters may not have realised that the lower portcullis stone (which sealed the chimney at a height of 393 cm) was a moveable stone, and since the only observable access above this height was the window (which began at 620 cm), it would make sense that the lower course stone was removed from the window to enable access for a person armed with a torch.

Another theory concerning the chimney was offered by Bob Moores, who argued that the chimney may have served as a "control datum for guiding the construction of the interior apartments" (Moores 2019: n.p.). Ahmed Fakhry, however, called the chimney "one of the unsolved problems of the pyramid" (Fakhry 1961: 102). Its positioning, he argued, certainly suggests that it carried significance, as it is placed at the very centre of the pyramid in Mustapha's survey, as well as in John S. Perring's, and terminates at ground level (Fakhry 1959: 68, fig. 33; Perring 1842: pl. xvi). The central point is an important position in a pyramid: it is from this point that the pyramid's perimeter was measured; the chamber in the neighbouring satellite pyramid was placed in the centre; and eventually, when completed, the capstone of the Bent Pyramid was placed 200 cubits directly above this point. Hans-Joachim Trumpp also points to the central positioning of the chimney, arguing that its placement may have carried a solar function "to provide a connection to the pyramid peak and to the Sun God Ra" (Trumpp 2014: 24).

THE IMPORTANCE OF THE SUMMER SOLSTICE

When the Bent Pyramid was being constructed, the perimeter was measured and aligned to the cardinal points. A huge pit was dug out of the earth and the lower chamber, the antechamber, and the chimney were built. The chimney may have been open to the elements for a period of time prior to the bulk of the pyramid being constructed over it (allowing time to complete the complex subterranean internals of the pyramid). If it was uncovered for a year, this is how

4 All measurements of the window are sourced from Maragioglio – Rinaldi (1964: tav. 12.1). Note that 620 cm is the height which Maragioglio and Rinaldi identify as "soglia originale", and 545 cm is the height of the window once the lower course stone was removed.

the sun would have interacted with the chimney at key moments: at the winter solstice, the midday sun would have shone 114 cm down the northern face of this shaft; at the equinoxes, the midday sun would have shone 269 cm down; at the summer solstice (and only on this date), the sun's light would have made it all the way to the bottom of this unusually long structure.[5]

On the summer solstice at Dahshur in 2550 BC, the sun's maximum elevation was 84.187°.[6] Its light would have shone 1503 cm down a vertical shaft with a north-south length of 153 cm. However, these dimensions are of the chimney at floor level; it is slightly greater at ground level; here, it is 158 cm (Maragioglio – Rinaldi 1964: tav. 12.2). The distribution of this length from the central axis is not shown by Maragioglio and Rinaldi; however, its width also increments (from 90 cm to 102 cm) and this increase is equally spread on either side of the east-west axis. I will assume that the same equal distribution about the north–south axis applies to its length.[7] This creates a triangle with a rise of 1527 cm and a run of 155.5 cm. The angle which the hypotenuse makes with the base is 84.185°. This is almost the exact angle of the sun on the summer solstice; the error is just 0.002°. Theoretically, therefore, on this date the dimensions of the chimney enabled the sun's light to create a dividing line which ran from the southern top of the chimney to the northern bottom on this date. Thus at noon on the summer solstice, the sun's light split this structure into two equal, but inverse triangles; one was filled with light, the other was dark (see fig. 1a).

The design of the chimney is dualistic; its general form is of a rectangular shaft with two stages of asymmetrical corbelling, in which the form of the bottom half is repeated in reverse in the top half. This two-fold design is echoed in the pyramid's own design. Its exterior carries two angles: the upper section is 43° 21'; the lower section is 54° 31' 13'' (Fakhry 1959: 67). Internally, the pyramid has two separate entrances leading to two different chambers. Taken together, the two-fold design of the chimney and of the pyramid hint that the division of the chimney into two equal portions of light and dark at the summer solstice was not a random accident of construction, but was similarly purposeful. Moreover, such an intended effect would help to clarify some of the other unusual dimensions of this shaft that otherwise escape easy explanation.

While we have no evidence that the chimney remained uncovered during the period of the summer solstice or that it was specifically designed to address solar measurements, the summer solstice sun shining down man-made structures like the chimney has been noted elsewhere in Egypt. It was known in Strabo's time that the noonday summer solstice sun shone directly down a nilometer at Syene (Strabo, 17. 1. 48). In the third century BC, Eratosthenes reputedly visited this well in order to assist him in his calculations of the earth's circumference. According to Hans Claude Hamilton and William Falconer, the construction of this well

5 The sun's light would have reached the floor of the passageway on other dates, but the summer solstice was the only date that the sun's light would have made it to the bottom of the chimney if the northern wall of the chimney extended all the way to the ground.

6 2550 BC is the date used in this article for solar observations. Note, however, that solar values shift very slowly and consequently, results will be unaffected by dates up to 50 years on either side. For example, at the summer solstice, the maximum elevation reached by the sun in 2500 BC was 84.184°; in 2550 BC, 84.187°; in 2600 BC, 84.190°. Elevations sourced from NASA's Jet Propulsion Laboratory (https://ssd.jpl.nasa.gov/horizons.cgi#top. Accessed 10th July 2020.).

7 Maragioglio and Rinaldi's west view of the chimney does not show this expansion favoring any side, so it is a safe assumption to distribute it evenly about the north-south axis (see Maragioglio – Rinaldi 1964: tav. 11.1).

"belonged to a very remote period" (Strabo, 17. 1. 48, n. 2). It is likely that this effect was known well before Eratosthenes, for he took the long journey south specifically to see it, implying that it was an established phenomenon. It is hard to imagine the ancient Egyptians not having noticed the way the sun illuminated the bottom of the well, because this nilometer would have been a hive of activity around the time of the summer solstice, since that was when the annual inundation was expected to arrive (Diodorus 1.36; Herodotus 2.19). Furthermore, this nome was believed to be the source of the Nile, so it would have been inspected daily for the first signs of the flood (Budge 1912: 120–22).

It is notable as well that the summer solstice appears to have been a date of importance for the Fourth Dynasty pyramid builders. Mark Lehner noted that from the eastern niche of the Sphinx temple, on the summer solstice, "the sun sets almost exactly midway between the Khufu and Khafre pyramids, thus construing the image of the *akhet*, 'horizon', hieroglyph" (Lehner 1985: 141). According to Juan Belmonte, the lower portion of the Bent Pyramid was designed to create a solar hierophany on the summer solstice (Belmonte – Magli 2015: 173–205). In Massimiliano Nuzzolo's opinion, "the Dahshur pyramids were mainly connected to the sun cult and the concept of the horizon as the place for the accomplishment of the sun cycle" (Nuzzolo 2015: 271). According to Lehner, the Bent Pyramid "probably had more to do with the solar cult and deification of the king than with any attempt at preserving his body" (Lehner 1997: 129).

The solar nature of the Bent Pyramid is certainly suggested by the length of its sides. William M. F. Petrie's survey determined the mean length of the sides to be 189.46 m, which he interprets as "360 cubits of 20.72 +/-.006, the ordinary Egyptian cubit" (Petrie 1887: 28, 31). According to Plutarch's account, the Egyptian calendar was originally 360 days long, then Thoth waged a bet with the moon and won five extra days (Plutarch 5.12). According to Kahl, this was the form of the civil calendar from at least the time of King Djoser (Kahl – Kloth – Zimmermann 1995: 70–71). According to Erik Hornung, these five days were often disregarded: "A tendency to regard the year as amounting to only 360 days is evident, for example when the daily income of a temple is stated to be one 360th of the yearly revenue. The well known disregard of the epagomenai in calendar schemes seems to be another consequence of this tendency" (Hornung – Krauss – Warburton 2006: 47). Thus we can see that the number of cubits comprising the length of the base of the Bent Pyramid matched the number of days in the year (excluding the five epagomenal days).[8]

All of this further supports the hypothesis that the chimney, like other parts of the pyramid, would have reflected the keen interest in astronomy held by the Fourth Dynasty Egyptians. This is further evidenced by the orientation of the pyramids of this time. The Bent Pyramid is a quarter of a degree off being precisely oriented to the compass points (Shaltout – Belmonte – Fekri 2007: 146). By the time his son's pyramid was constructed (the Great Pyramid), this error was reduced to 3' 6'' (Lehner 1997: 108). To achieve this level of accuracy required a thorough and ongoing study of either the sun or the northern polar stars (Belmonte 2001: 1–3).

8 There was another structure dating to the New Kingdom which used the same system wherein each cubit represented one day, evident in Diodorus' account of the tomb of King Ramses II: "... there is a circular border of gold crowning the monument, three hundred and sixty-five cubits in circumference and one cubit thick; upon this the days of the year are inscribed, one in each cubit of length..." (Diodorus 1. 49. 5–6).

In either case, this suggests a dedication to astronomy, which is unlikely to be restricted to one section of the skies and not the other.

USING THE CHIMNEY TO DETERMINE SOLAR ELEVATIONS ON OTHER DATES

Assuming that the chimney was open to the elements before the construction of the pyramid, and that the entry of the noon-day sun's light into the chimney was unimpeded, then a specific light effect would have occurred within the chimney which was only in harmony with the dimensions of the chimney for one brief moment in the year. At noon on the summer solstice, the sun's light would have made it to the very bottom of the chimney and this would have been the only date that it did so if the northern wall of the chimney extended all the way to the floor. However, the entrance into the chimney is at its base and on its northern side, so currently there is a 393 cm high opening on that side which allowed light on dates either side of the solstice to reach the floor of the passageway that links the chimney to the lower chamber. This enabled the sun's maximum elevations to be accurately and easily measured on dates around the summer solstice, which otherwise would have been extremely difficult to measure on account of these elevations shifting by small fractions of a degree each day at this time of year.

Working out what elevation the sun had to carry for its rays to pass below the lintel of this doorway will make it possible to work out all the dates that the sun's rays hit the floor of the passageway. The doorway is 393 cm high and so the length of the north wall of the chimney is 1527 cm - 393 cm = 1134 cm. These dimensions create an angle of 82.192°, so if the sun were at an elevation which exceeded this angle, its light would pass beneath the lintel and reach the floor (see fig. 1c). According to NASA's Jet Propulsion Laboratory, the sun's elevation at Dahshur in 2550 BC exceeded this angle from 1st June to 17th July (Gregorian). If the surface opening of the chimney was open and there was an unimpeded line of sight to the midday sun running southwards at an angle of 82.192° from the horizon, then for a total of 47 days, the sun's light would have passed beneath the lintel and cast its light onto the floor of the passageway.[9] Tab. 1 lists all the dates in 2550 BC that the light of the midday sun hit the floor of the passageway, along with corresponding solar elevations, as well as the distance of this line of light from the chimney's virtual north wall.[10]

From the winter solstice onwards, the midday light of the sun crept down the north wall of the chimney. On 1st June, its light passed beneath the door lintel and reached the floor for the first time in the year, creating a line of white light on the floor of the passageway that was 50.7 cm beyond the virtual north wall of the chimney.[11] On 2nd June, this line of light was

9 There are remnants of a false ceiling in this passageway which Maragioglio and Rinaldi measured as 45 cm, thus lowering the height of the lintel to 348 cm (see Maragioglio – Rinaldi 1964: tav. 11.1). It is not known if this was original or a later addition to shore up the lintel. If it was original, then its presence reduced the angle required for the sun's light to pass below the lintel to 82.48°. This removes two days at the start and at the end of the effect, reducing the total number of days that the sun's light made it to the floor to 43.

10 The "virtual" north wall is where the north wall would terminate if it continued to the floor (*i.e.* if the doorway did not interrupt the wall).

11 This line of light was parallel to the southern wall of the chimney.

46.3 cm from the north wall. On 3rd June, it was 42.3 cm from the wall. With each passing day, the line of light approached the point where the north wall of the chimney would be if it had continued to the floor. By the summer solstice (24th June), this line was nigh on perfect; it was 0.043 cm within the chimney.[12] It may have been that the intention was for the sun's line of light to reach the nexus point where the chimney ends and the passageway begins, thus in some symbolic manner completing the chimney (see fig. 1a). After the solstice, the line of light began retreating, moving further away from the southern wall with each passing day. By 17th July, it was 52 cm away from it. On the following day, the sun's rays were impeded by the lintel and the light effect ceased to occur on the floor of the passageway. Instead, the sun's light crept up the north face of the chimney once more, not to reach the floor again until the first day of June in the following year. This moving line of light offered the Egyptians a simple method for accurately measuring the incredibly tricky solar elevations over the summer period. For 47 days, the sun would have cast its light all the way to the ground, unimpeded by the northern wall of the chimney. What resulted was a clear, straight line upon the floor of the passageway. This made it easy to measure its distance from the south wall of the chimney on each of these days. In doing so, knowing full well the height of the chimney, the Egyptians could determine the angle of the sun.

The effects mentioned in this article require the surface opening of the chimney to not have been built over for one summer and for an unimpeded line of sight running southwards at an angle of 82° from the horizon to have been retained for this time. The builders may have delayed construction of the lower courses of the pyramid until one summer had passed in order to enable the chimney to be used for the solar observations proposed in this article. It might seem unusual to build such a finely tuned solar measurement device and then to build over it, however the sun's path repeats each year without noticeable variation in the short term, so only one summer of solar observations was required to record the data listed in tab. 1; there was no need to measure it again over the following summer. Thereafter, this "device" was encased in the exact centre of the pyramid, perhaps as a means of preserving a solar link within the heart of the structure in keeping with the pyramid's solar inspired dimensions.

RHIND MATHEMATICAL PAPYRUS

The Rhind Mathematical Papyrus shows how the ancient Egyptians deduced the angle of a triangle; they determined the ratio of a triangle's rise to its run (known as a *sḳd*). While the Rhind Papyrus was written in the Second Intermediate Period, parts of it were copied from an older text that dates to the Middle Kingdom (Imhausen 2020: 64–65). However, Problems 56–59 deal with pyramid slopes and since such formulas were required for the construction of pyramids, it is likely that this was the mathematical approach used to determine the slopes of Old Kingdom pyramids. In support of this, Luca Miatello argues that Problem 60 may reference the slope and method of construction of the Third and Fourth Dynasty pyramids (Miatello 2009: 156). Furthermore, Miatello notes that the *sḳd* of 5 palms and 1 finger appears

12 The summer solstice is the date that the sun reaches its maximum elevation for the year. According to NASA's Jet Propulsion Laboratory, this occurred on 15th July (Julian) in 2550 BC, which is equivalent to 24 June (Gregorian) (https://ssd.jpl.nasa.gov/horizons.cgi#top. Accessed 10th July 2020.).

Date (Gregorian)	Apparent solar elevation	Distance of sun's line of light north of chimney north wall (cm)
1 June	82.31°	+ 50.7
2 June	82.47°	+ 46.3
3 June	82.62°	+ 42.3
4 June	82.76°	+ 38.5
5 June	82.9°	+ 34.7
6 June	83.03°	+ 31.2
7 June	83.15°	+ 27.9
8 June	83.27°	+ 24.7
9 June	83.38°	+ 21.7
10 June	83.48°	+ 19
11 June	83.58°	+ 16.3
12 June	83.67°	+ 14
13 June	83.75°	+ 11.7
14 June	83.83°	+ 9.6
15 June	83.9°	+ 7.8
16 June	83.96°	+ 6.1
17 June	84.01°	+ 4.5
18 June	84.06°	+ 3.4
19 June	84.1°	+ 2.3
20 June	84.13°	+ 1.5
21 June	84.16°	+ 0.7
22 June	84.17°	+ 0.4
23 June	84.18°	+ 0.1
24 June	84.187°	0
25 June	84.18°	+ 0.1
26 June	84.17°	+ 0.4
27 June	84.16°	+ 0.7
28 June	84.13°	+ 1.5
29 June	84.1°	+ 2.3
30 June	84.05°	+ 3.6
1 July	84°	+ 5
2 July	83.95°	+ 6.3
3 July	83.89°	+ 7.9
4 July	83.82°	+ 9.8
5 July	83.74°	+ 12

Date (Gregorian)	Apparent solar elevation	Distance of sun's line of light north of chimney north wall (cm)
6 July	83.66°	+ 14.2
7 July	83.57°	+ 16.6
8 July	83.47°	+ 19.3
9 July	83.36°	+ 22.3
10 July	83.25°	+ 25.2
11 July	83.13°	+ 28.5
12 July	83°	+32
13 July	82.87°	+ 35.5
14 July	82.73°	+ 39.3
15 July	82.58°	+ 43.4
16 July	82.42°	+ 47.7
17 July	82.26°	+ 52

Tab. 1 List of all the dates in 2550 BC that the light of the midday sun hit the floor of the passageway, along with corresponding solar elevations, and the distance from the chimney's virtual north wall

in four problems in the Rhind Papyrus, and that was "a typical characteristic of Old Kingdom pyramids and is not attested after Pepi II" (Miatello 2009: 157). According to Gay Robins and Charles Shute, the origins of the Rhind Papyrus date back to at least the time of the Old Kingdom pyramid builders (Robins – Shute 1987: 58).

The techniques found in the Rhind Papyrus (specifically Problem 56) can be used to show how the data produced from the light effect in the chimney could be used to determine apparent elevations of the sun (Chace 1927: 96–97). Take 12th June as an example. The rise and run of the triangle are 1527 cm and 169.5 cm respectively.[13] To determine the *sḳd*, firstly these numbers have to be converted into cubits. The height of the chimney is 1527 cm, which is 29 cubits of 52.655 cm.[14] The triangle takes the form of 1527 cm ÷ 52.655 cm = 29 cubits for the rise; and 169.5 cm ÷ 52.655 cm = 3.219 cubits for the run. The Rhind Papyrus solves this problem by multiplying 29 to get 3.219. The method is to add fractions of 29 to achieve this goal.

The fractions involved here are: 1/10 = 2.9; 1/100 = 0.29; 1/1000 = 0.029.

1/10 + 1/100 + 1/1000 = 2.9 + 0.29 + 0.029 = 3.219.

Multiplying each fraction by 7 yields the equivalent measure in palms. These are then added together to reveal the *sḳd* value: 7/10 + 7/100 + 7/1000 = 777/1000 palms per cubit.

To test how accurate this result is, these measures have to be converted into modern measures. There are 7 palms per cubit. The cubit in use here is the chimney cubit (52.655 cm), so the value of the palm is 7.522 cm. The run is 0.777 palms, which is 5.845 cm. The rise is 29

13 The line of light on 12th June was 167 cm from the south wall at the base of the chimney; however, the run is determined by the length of the chimney at its top, which is 2.5 cm greater than its length at the base; hence the run used for calculations here is 169.5 cm.

14 1527 ÷ 29 = 52.655, so the cubit in use in the chimney is 52.655 cm.

cubits, which is 1527 cm. This triangle carries an angle of 83.67°, which is exact. This method, while cumbersome, does yield the correct angle for any right-angled triangle.

CONCLUSION

While it is currently impossible to verify with on-site experimentation, or through written documentary evidence, this preliminary analysis has shown that it is entirely possible that the chimney in the Bent Pyramid provided a mechanism by which its builders could measure the sun's maximum elevations during the summer. This might also provide an explanation for the unusual dimensions of the chimney, which may have been determined by the elevation of the sun at noon on the summer solstice, a date on which the sun's light would have reached the bottom of the chimney and divided the structure into two equal parts; one filled with light, the other dark.

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INTERNET RESOURCES

NASA's Jet Propulsion Laboratory Horizons Web-Interface (https://ssd.jpl.nasa.gov/horizons.cgi#top)

Kyle Weyburne
independent scholar; roboticafactory@gmail.com